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Quantum Superdense Coding Based on Coherent States in Cavity QED

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Abstract A physical scheme for the implementation of quantum superdense coding has been proposed in Cavity QED. The detuned interaction between Λ-type three-level atoms and coherent fields constitute the main superdense coding process. The quantum superdense coding can be realized in an easier way, and the atoms are not excited during the whole process, so the effect of atomic decay is eliminated naturally.

Keywords Quantum superdense coding · Quantum entanglement · Cavity QED · Coherent states

Quantum entanglement is at the core part of quantum information theory. It finds many intriguing applications in quantum information processing, which do not have the classical counterparts, such as quantum teleportation [1], quantum cryptography [2], quantum superdense coding [3], and quantum remote preparation [4] etc. In Quantum superdense coding, it is possible to send two classical bits of information by sending only one single qubit [3]. Here the prior shared entanglement is a necessity for superdense coding. It is this prior shared entanglement that makes the classical communication capacity enhanced. Recently, the standard superdense coding process has been generalized in many different directions, such as superdense coding with multi-particle users [5–10], superdense coding in continuous variables system [11–15]. Because in the original superdense coding protocol, the prior shared entangled state must be maximally entangled ones, so that the sender can faithfully

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send 2 bits of classical information by sending one qubit. Many works show that if the prior shared entanglement is not maximal, the success probability will be lower than 1.0. But if it succeeds, 2 bits of classical information have been transmitted [16–19]. In another way, the optimal superdense coding with some error has been proposed very recently [20]. Superdense coding also has been studied by the assistance of mixed entanglements [21]. This also leads to the investigation on the relationship between the prior entanglement and the maximal number of alphabets which can be perfectly transmitted deterministically [22–24]. Most of the previous contributions are based on the two-state qubit system. Very recently, more attention has been paid on the superdense coding based on the higher dimensional systems [25, 26].

Although, Superdense coding has been developed much from theoretical respect. But, up to now, superdense coding has been experimentally realized only in optical system, nuclear magnetic resonance and Cavity QED system [11–15, 27–29]. It is known that cavity QED is another promising candidate for realizing quantum information processing [30]. Compared with photons, atomic systems have a relatively long lifetime, which permits to complete enough operations before the decay of atoms. In cavity QED, the cavity decay is a main obstacle to the experimental realization. So Zheng et al. proposed a non-resonant scheme to eliminate the effect of cavity decay. Here the cavity mode is only virtually excited [31]. Along this line, Lin et al. proposed a superdense coding scheme by using this kind of non-resonant interaction between atoms and cavity mode [32]. Although this scheme is insensitive to cavity decay, it is still sensitive to thermal field. Flowing Zheng's proposal [33], Ye et al. proposed another superdense coding scheme, in which the non-resonant interaction between atom and cavity mode has been assisted by a strong driving field. So, the photon-number-dependent parts are cancelled in the effective Hamiltonian, and the scheme is insensitive to both the cavity decay and the thermal field. Very recently, the atomic entanglement has been created in lab [34], and the teleportation of unknown atomic states also has been experimentally realized in ion traps [35, 36]. Although the quantum dense coding for atomic states has been demonstrated [29], the implementation process is rather complex. So it is necessary to find some simple way for realizing the quantum superdense coding for atomic states.

In the above-mentioned theoretical superdense coding schemes, only the effect from cavity has been studied. Actually, the decay of atom is also an important aspect which we must take into account. Furthermore, the preparation of vacuum state or some Fock state field is not easy in experiment. In view of these two points, Yang et al. proposed a quantum information processing scheme by using the interaction (non-resonant) between atoms and coherent field [37]. Here, the two degenerate ground states are used as logical states, and the interaction is a non-resonant one. So the ground states are not excited during the interaction, which eliminates the effect of atomic decay on the scheme. In addition, it is easier to prepare the coherent state than the Fock state. Following the idea of Yang's proposal we propose one superdense coding scheme by using coherent states in Cavity QED.

Let's consider the interaction between a Λ-type three-level atom and a coherent field. The level structure of the atoms is depicted in Fig. 1, where $|e\rangle$and $|g\rangle$ are two degenerate ground states, and $|i\rangle$ is the excited state. Here, there is a large detuning between the atomic transition ($|e\rangle \leftrightarrow |i\rangle$) frequency ω_0 and the frequency of coherent field ω_c, i.e. $\Delta = \omega_c - \omega_0$. If the detuning is large enough, the excited state $|i\rangle$ can be adiabatically eliminated during the interaction and the effective Hamiltonian can be expressed as follow [38]:

$$\hat{H} = -\lambda a^{+} a(|e\rangle\langle g| + |g\rangle\langle e|) - a^{+} a(\beta_1 |e\rangle\langle e| + \beta_2 |g\rangle\langle g|) \tag{1}$$

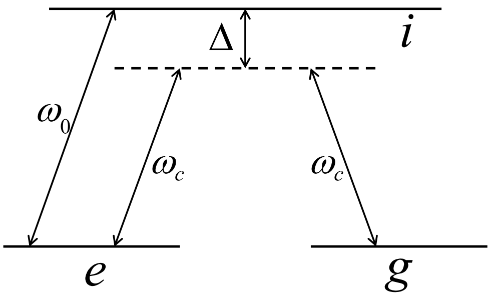

Fig. 1 Level configuration of the atoms used in the scheme

where $\lambda = g_1 g_2/\Delta$, $\beta_1 = g_1^2/\Delta$, $\beta_2 = g_2^2/\Delta$, g_1, g_2 being the coupling constant between the cavity mode and the transitions $|i\rangle \to |e\rangle, |i\rangle \to |g\rangle$ respectively. Suppose $g = g_1 = g_2$, $\lambda = \beta_1 = \beta_2 = g^2/\Delta$.

Governed by the above effective Hamiltonian, the state of the system will evolve in the following way:

$$|e\rangle|\alpha\rangle \xrightarrow{U(t)} (1/2)[(|\alpha\rangle + |\alpha e^{2i\lambda t}\rangle)|e\rangle - (|\alpha\rangle - |\alpha e^{2i\lambda t}\rangle)|g\rangle] \tag{2a}$$

$$|g\rangle|\alpha\rangle \xrightarrow{U(t)} (1/2)[(|\alpha\rangle + |\alpha e^{2i\lambda t}\rangle)|g\rangle - (|\alpha\rangle - |\alpha e^{2i\lambda t}\rangle)|e\rangle] \tag{2b}$$

From the discussions in Ref. [37] we can say $|\alpha_+\rangle = (1/\sqrt{2})(|\alpha\rangle + |-\alpha\rangle)$, $|\alpha_-\rangle = (1/\sqrt{2})(|\alpha\rangle - |-\alpha\rangle)$ are two orthogonal basis for big $|\alpha|$ (for example, $|\alpha| \geq 3$). Now, if we select velocity of the atom appropriately, we can let the interaction time satisfy $t = \pi/(2\lambda)$, thus:

$$|e\rangle|\alpha\rangle \xrightarrow{\lambda t=\pi/2} (1/\sqrt{2})[|\alpha_+\rangle|e\rangle - |\alpha_-\rangle|g\rangle] \tag{3a}$$

$$|g\rangle|\alpha\rangle \xrightarrow{\lambda t=\pi/2} (1/\sqrt{2})[|\alpha_+\rangle|g\rangle - |\alpha_-\rangle|e\rangle] \tag{3b}$$

$$|e\rangle|-\alpha\rangle \xrightarrow{\lambda t=\pi/2} (1/\sqrt{2})[|\alpha_+\rangle|e\rangle + |\alpha_-\rangle|g\rangle] \tag{3c}$$

$$|g\rangle|-\alpha\rangle \xrightarrow{\lambda t=\pi/2} (1/\sqrt{2})[|\alpha_+\rangle|g\rangle + |\alpha_-\rangle|e\rangle] \tag{3d}$$

Next, we will discuss the superdense coding process in more detail.

First, let one atom through one coherent field. Here, the atom is in state $|e\rangle$ and the interaction time can be definitely set to be $t = \pi/(2\lambda)$. So the evolution of the system state is governed by (3a), and the state of the system is:

$$|\Phi\rangle = (1/\sqrt{2})[|\alpha_+\rangle|e\rangle - |\alpha_-\rangle|g\rangle] \tag{4}$$

Suppose that the atom and the coherent field have been distributed to Alice and Bob respectively. Now, Alice will perform on her atom any one of the four local operations $\{I, \sigma_x, i\sigma_y, \sigma_z\}$to encode two bits of classical information. Depending on the operations, the distributed entangled state will be transformed into the following four different states, respectively:

$$|\Phi^I\rangle = I \otimes I|\Phi\rangle = (1/\sqrt{2})[|\alpha_+\rangle|e\rangle - |\alpha_-\rangle|g\rangle] \tag{5a}$$

$$|\Phi^{\sigma_x}\rangle = I \otimes \sigma_x|\Phi\rangle = (1/\sqrt{2})[|\alpha_+\rangle|g\rangle - |\alpha_-\rangle|e\rangle] \tag{5b}$$

$$|\Phi^{i\sigma_y}\rangle = I \otimes i\sigma_y|\Phi\rangle = (1/\sqrt{2})[|\alpha_+\rangle|g\rangle + |\alpha_-\rangle|e\rangle] \tag{5c}$$

$$|\Phi^{\sigma_z}\rangle = I \otimes \sigma_z|\Phi\rangle = (1/\sqrt{2})[|\alpha_+\rangle|e\rangle + |\alpha_-\rangle|g\rangle] \tag{5d}$$

Then, Alice will send her atom to Bob. If Bob can learn the state (anyone of the above four possible states) of the two atoms by local operations and measurement, then he can extract the two bits classical information deterministically. To realize this discrimination process, Bob will let the atom through the coherent field, and the interaction time is also set to be $t = \pi/(2\lambda)$. This interaction is also governed by Hamiltonian in (1) and the four states will undergo different evolutions:

$$|\Phi^I\rangle \to |\alpha\rangle|e\rangle \tag{6a}$$

$$|\Phi^{\sigma_x}\rangle \to |\alpha\rangle|g\rangle \tag{6b}$$

$$|\Phi^{i\sigma_y}\rangle \to |-\alpha\rangle|g\rangle \tag{6c}$$

$$|\Phi^{\sigma_z}\rangle \to |-\alpha\rangle|e\rangle \tag{6d}$$

To discriminate the four states, Bob only needs to detect the state of atom and the coherent field, respectively. Thus, Bob receives the two bits classical information which is encoded by sender (Alice).

Next, we will discuss the experimental feasibility of the current scheme. Here, because the atomic levels we are using are the two degenerate ground levels, the atomic decay will not affect the scheme. We only need to consider the effect of cavity decay of coherent field. From the discussion in Ref. [37], the interaction time required for completing the whole superdense coding process is about $t = 1.0 \times 10^{-3}$ s for some typical parameters of the system (for example, $g = 2\pi \times 25$ kHz, and the detuning $\Delta = 100$ g for satisfying the large detuning condition), the decoherence time of the coherent field is $t_{deco} = T_{cavity}/\bar{n} = 4.0 \times 10^{-2}$ s with $\bar{n}$ being the mean photon number of the field, and T_{cavity} is the lifetime for the cavity [39]. So $t_{deco} > t$, and it is possible to realize the scheme within the current technology. The detection of the atomic states can be realized by using the ionization techniques [30]. Yurke et al. have proposed the method for detecting the coherent field and for distinguishing state $|\alpha\rangle$ from $|-\alpha\rangle$ [40].

In summary, we presented a scheme for realizing quantum superdense coding in Cavity QED, which is mainly based on the detuned interaction between Λ-type three-level atoms and coherent fields. Because of the special feature of this kind of interaction, the four different entangled states after the four operations (encoding the two bits of classical information) can be discriminated easily. The current scheme inheres the following advantages: (i) it is very simple, and the whole process can be realized by single-step interaction; (ii) the atomic decay doesn't affect the scheme because the atoms are all in their ground states during the whole process; (iii) the interaction is a non-resonant one, so the requirements on the interaction system are greatly loosened.

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